The Influence of Water Depth on the Hydroelastic Response of a Very Large Floating Platform

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Abstract

The hydroelastic response of a two-dimensional very large floating platform (VLFP) to plane incident wave is investigated for three different cases: infinite, finite and shallow water depth. An integro-differential equation is presented to describe the deflection of the platform due to incident waves. Reflection and transmission coefficients are obtained as well. We consider the case of a strip and a half-plane. Numerical results are obtained for various values of the parameters. The results for the strip and for the semi-infinite platform are compared for different values of depth.

1 Introduction

The study of the behavior of floating flexible plates on waves obtains great interest. This problem is important thanks to the investigation of the interaction between large floating platforms (airports etc.) or ice fields and surface waves. The thickness of the floating objects compared to horizontal parameters is small and they are modeled as thin elastic plates.

Recently HERMANS [1] derived an exact integral-differential equation for the deflection of a VLFP at deep water. The equation was solved numerically by means of a boundary element method and a mode expansion. Later HERMANS

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in [2] and [3] used this formulation to derive boundary condition to apply the 'ray method' for short wave diffraction.

The short wave expansion of TAKAGI et al. [4] leads to similar results. They used an eigenfunction expansion method.

The use of the integral-differential equation is more flexible to derive asymptotic results than the set of equatins of KHABAKHPASHEVA and KOROBKIN [5].

TKACHEVA [6] solved this problem by using the Wiener-Hopf technique. The shallow water problem was solved by STUROVA [7]. She used the boundary integral equations. KIM and ERTEKIN [8] applied eigenfunction-expansion method for solving this problem.

Here we study the diffraction of surface waves by large floating flexible platform (FFP) of general geometric form which floats on the surface of the incompressible fluid of infinite (IWD), finite (FWD) and shallow (SWD) water depth. Differences of these three cases will be indicated in the paper.

We solve the problem for oblique incident waves (including perpendicular waves) for two different forms of the platform: an infinitely long strip of finite width and a semi-infinite plate. For both forms results are obtained and compared. Reflection and transmission of incoming waves are investigated.

2 Formulation of the problem

The mathematical formulation is derived for the diffraction of waves by FFP which floats at the surface of an ideal incompressible fluid of depth h which is varied for different cases. Incoming short waves propagate from the open fluid (in positive x-direction). We assume waves in still water and introduce the velocity potential $\nabla\Phi(x,y,z,t) = \vec{V}(x,y,z,t)$. $\Phi(x,y,z,t)$ is a solution of the Laplace equation

$$\Delta\Phi = 0 \tag{1}$$

in the fluid ($z < 0$) together with the conditions:
at the bottom $z = -h$ (we use this condition for finite and shallow water)

$$\frac{\partial\Phi}{\partial z} = 0 \tag{2}$$

and surface conditions at $z = 0$

$$\frac{\partial\Phi}{\partial z} = \frac{\partial w}{\partial t} \text{ when } x \in \mathcal{P} \text{ and } \frac{\partial\Phi}{\partial z} = -\frac{1}{g}\frac{\partial^2\Phi}{\partial t^2} \text{ when } x \in \mathcal{F} \tag{3}$$

where $w(x, y, z, t)$ denotes the free surface elevation under the platform. Here and below in case of the strip of width l of infinite length $0 \leq x \leq l$, $-\infty < y < \infty$ we define the fluid area $-\infty < x < 0 \cup l < x < \infty$ as $\mathcal{F}$, the platform area $0 < x < l$ as $\mathcal{P}$ and the dividing surface $x = 0 \cup x = l$ as $\mathcal{S}$ (this is shown in figure 1); whereas for the semi-infinite platform (SIP) $0 \leq x < \infty$, $-\infty < y < \infty$ respectively $\mathcal{F}$ is $x < 0$, $\mathcal{P}$ is $x > 0$ and $\mathcal{S}$ is $x = 0$.

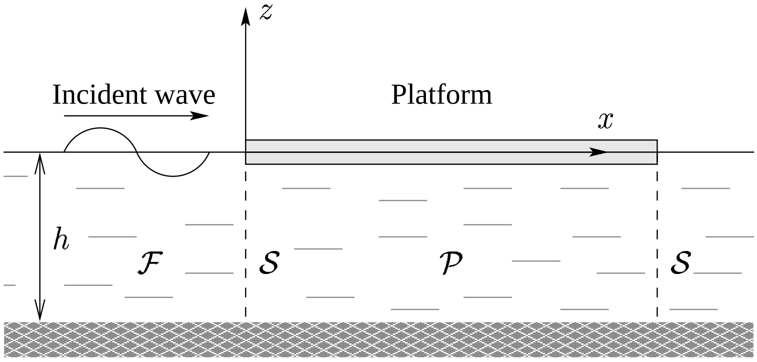

Fig.1 Definition sketch of the problem

The platform is assumed to be a thin layer at the free surface $z = 0$, which seems to be a good model for a shallow draft platform which is modeled then as an elastic plate with zero thickness. To describe the deflection of the platform w we apply the thin plate theory, which leads to a differential equation in the following form:

$$D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w + m\frac{\partial^2 w}{\partial t^2} = P(x, y, z, t) \tag{4}$$

at $z = 0$ for the platform area $x \in \mathcal{P}$, where m is the mass of unit area of the platform, D is its equivalent flexural rigidity, P is the linearized pressure

$$P = -\rho\frac{\partial \Phi}{\partial t} - \rho g w \text{ at } z = 0 \tag{5}$$

here ρ is the density of the water.

For infinite and finite water depth cases we apply the operator $\partial/\partial t$ to (4) and use the surface condition and (5) to arrive at the following equation for Φ at $z = 0$ in the platform area $(x, y) \in \mathcal{P}$:

$$\left\{\frac{D}{\rho g}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 + \frac{m}{\rho g}\frac{\partial^2}{\partial t^2} + 1\right\}\frac{\partial \Phi}{\partial z} - \frac{1}{g}\left\{\frac{\partial^2}{\partial t^2}\right\}\Phi = 0 \tag{6}$$

The free edges of the platform are free of moment and shear force, boundary conditions are:

$$\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2} = 0 \text{ and } \frac{\partial^3 w}{\partial x^3} + (2 - \nu)\frac{\partial^3 w}{\partial x \partial y^2} = 0 \text{ when } x, y \in \mathcal{S} \tag{7}$$

where ν is Poisson's ratio.

For shallow water we have also the approximate transition conditions at the edges of the platform

$$\Phi_n \text{ and } \Phi_t \text{ continuous at } \mathcal{S} \tag{8}$$

where n is the normal to the edge of platform. Physically these conditions express that the mass of the water is conserved and the energy flux is continuous.

The harmonic wave can be written in the following form

$$\Phi(x,y,z,t) = \phi(x,y,z)e^{-i\omega t} \tag{9}$$

The incident wave for the fluid of infinite depth equals

$$\phi^{inc} = -\frac{igA}{\omega}e^{ik_0(x\cos\beta+y\sin\beta)+k_0 z} \tag{10}$$

where A is the wave height, ω is the frequency and k_0 is the wave number. $k_0 = \omega^2/g$ for IWD and $k_0 = \omega/\sqrt{gh}$ for the fluid of shallow depth when incident wave is written as

$$\phi^{inc} = -\frac{igA}{\omega}e^{ik_0(x\cos\beta+y\sin\beta)} \tag{11}$$

For finite water incident waves equals

$$\phi^{inc} = -\frac{\cosh k_0(z+h)}{\cosh k_0 h}\frac{igA}{\omega}e^{ik_0(x\cos\beta+y\sin\beta)} \tag{12}$$

where the wave number obeys the dispersion relation $k_0 \tanh k_0 h = K$, here $K = \omega^2/g$. Length of incoming waves is $\lambda = 2\pi/k_0$.

3 Finite Water Depth

In this chapter we derive the solution for finite water for the strip and for the semi-infinite platform cases.

The fluid domain is split up into two regions with the interface $\mathcal{S}$. The potential function in $\mathcal{F}$ is written as a superposition of the incident wave potential and a $\phi_{dis}(\vec{x})$, which is the sum of classical diffraction potential and radiation potential, as follows

$$\phi^{\mathcal{F}}(\vec{x}) = \phi^{inc}(\vec{x}) + \phi^{dis}(\vec{x}) \tag{13}$$

while the total potential in $\mathcal{P}$ is denoted by $\phi^{\mathcal{P}}$. It will be shown that this choice leads to an interesting way to derive an integral equation. At the dividing surface $\mathcal{S}$ we require continuity of the total potential and its normal derivative.

We introduce the Green's function $\mathcal{G}(\vec{x},\vec{\xi})$ that fulfills $\Delta\mathcal{G} = 4\pi\delta(\vec{x}-\vec{\xi})$, the free surface and the bottom and the radiation condition. We apply Green's theorem to the potential $\phi^{\mathcal{F}}$ and $\phi^{\mathcal{P}}$ respectively. For $x, y \in \mathcal{F}$ we have:

$$4\pi\phi^{dis} = -\int\limits_{\mathcal{S}\cup\mathcal{F}} \left(\phi^{dis}\frac{\partial\mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial\phi^{dis}}{\partial n}\right) dS;\; 0 = \int\limits_{\mathcal{S}\cup\mathcal{P}} \left(\phi^{\mathcal{P}}\frac{\partial\mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial\phi^{\mathcal{P}}}{\partial n}\right) dS \quad (14)$$

and in the region $x, y \in \mathcal{P}$:

$$0 = -\int\limits_{\mathcal{S}\cup\mathcal{F}} \left(\phi^{dis}\frac{\partial\mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial\phi^{dis}}{\partial n}\right) dS;\; 4\pi\phi^{\mathcal{P}} = \int\limits_{\mathcal{S}\cup\mathcal{P}} \left(\phi^{\mathcal{P}}\frac{\partial\mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial\phi^{\mathcal{P}}}{\partial n}\right) dS \quad (15)$$

The integrals over $\mathcal{F}$ become zero, due to the zero current free surface condition for $\mathcal{G}$ and ϕ^{dis}. We add up the two expressions in (15) and using the free surface condition for the Green's function and the potential ϕ^{dis} leads us to

$$4\pi\phi^{\mathcal{P}} = \int\limits_{\mathcal{S}} \left(\left[\phi\right]\frac{\partial\mathcal{G}}{\partial n} - \mathcal{G}\left[\frac{\partial\phi}{\partial n}\right]\right) dS + \int\limits_{\mathcal{P}} \left(K\phi^{\mathcal{P}} - \phi^{\mathcal{P}}_{\zeta}\right)\mathcal{G}\, dS \text{ for } x, y \in \mathcal{P} \quad (16)$$

where we used the notation $\left[...\right]$ for the jump of the concerned function. Furthermore we use the jump condition between the potentials ϕ^{dis} and $\phi^{\mathcal{P}}$ and their normal derivatives. For the total potential the jumps are zero and then we obtain

$$4\pi\phi^{\mathcal{P}} = \int\limits_{\mathcal{S}} \left(\phi^{inc}\frac{\partial\mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial\phi^{inc}}{\partial n}\right) dS - \int\limits_{\mathcal{P}} \left(\mu\phi^{\mathcal{P}}_{\zeta} - \mathcal{D}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right)^2 \phi^{\mathcal{P}}_{\zeta}\right)\mathcal{G}\, dS \quad (17)$$

where we have used relation (6) for $\phi^{\mathcal{P}}$ and introduced parameters $\mathcal{D} = D/\rho g$, $\mu = m\omega^2/\rho g$. Relation (17) is suitable for further manipulation to end up with a differential-integral equation, that can be solved numerically. The Green's function itself has a weak singularity, so we may take the limit $z \to 0$ and use (6) to express $\phi^{\mathcal{P}}$ in terms of an operator acting on $\phi^{\mathcal{P}}_z$. Furthermore we notice that the first integral on the right-hand side of (17) can be simplified significantly. This term is independent of the parameters of platform, hence it is the same if there is no platform present. Therefore it equals $4\pi\phi^{inc}$. This also can be verified by manipulating the integrals.

$$4\pi\left(\phi_z^{\mathcal{P}} - \mu\phi_z^{\mathcal{P}} + \mathcal{D}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \phi_z^{\mathcal{P}}\right) +$$
$$K\int_{\mathcal{P}}\left(\mu\phi_\zeta^{\mathcal{P}} - \mathcal{D}\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right)^2 \phi_\zeta^{\mathcal{P}}\right)\mathcal{G}\, dS = 4\pi\phi_z^{inc} \tag{18}$$

which is valid at $z = 0$.

Taking into account (3) for the deflection of the platform w we obtain the following equation

$$\left(\mathcal{D}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 - \mu + 1\right) w(x, y) =$$
$$\frac{K}{4\pi}\int_{\mathcal{P}}\left(\mathcal{D}\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right)^2 - \mu\right) w(\xi, \eta)\mathcal{G}(x, y; \xi, \eta)\, d\xi d\eta + Ae^{ik_0 x\cos\beta} \tag{19}$$

The Green's function obeys the boundary conditions at the free surface and at the bottom and radiation condition, see HERMANS [3]. It has the form

$$\mathcal{G}(x, y; \xi, \eta) = -2\int_{\mathcal{L}'} \frac{k\cosh kh}{k\sinh kh - K\cosh kh} J_0(kr)\, dk \tag{20}$$

at $z = 0$ where $\mathcal{L}'$ is contour of integration in the complex k-plane and given in figure 2, from 0 to $+\infty$ underneath the singularity $k = k_0$, chosen for fulfilling of the radiation condition, $J_0(kr)$ - Bessel's function, while r is horizontal distance, so $r = \sqrt{(x-\xi)^2 + (y-\eta)^2}$.

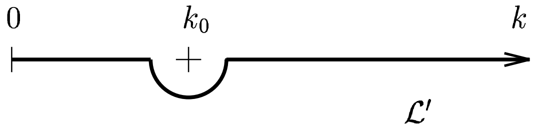

Fig.2 Contour of integration

As first case the **strip** is considered. The deflection of the platform is represented as a superposition of exponential function in the following form

$$w(x, y) = \sum_n \left(a_n e^{i\kappa_n x} + b_n e^{-i\kappa_n x}\right) e^{ik_0 y\sin\beta} \tag{21}$$

at $z = 0$ for $0 \le \beta \le \pi/2$ where 'amplitudes' a_n, b_n and reduced wave numbers κ_n will be determined. For the description of the connection between platform deflection and potential we use first condition of (3). Later we will see that inhomogeneous term behaves like $e^{ik_0 x}$ does not indicate that the solution behaves accordingly.

We insert the Green's function in (19). To carry out integration with respect to η we use the Sonine-Gegenbauer expression for Bessel's function $J_0(kr)$:

$$\int_0^\infty \cos(bt) J_0\left(k\sqrt{a^2+t^2}\right) dt = \begin{cases} 0 & \text{if } k < b \\ \frac{\cos a\sqrt{k^2-b^2}}{\sqrt{k^2-b^2}} & \text{if } k > b \end{cases} \tag{22}$$

Now we integrate with respect to ξ and obtain

$$\sum_n \left(\mathcal{D}\kappa^{(n)^4} - \mu + 1\right)\left(a_n e^{i\kappa_n x} + b_n e^{-i\kappa_n x}\right) = i\sum_n \frac{K}{2\pi}\left(\mathcal{D}\kappa^{(n)^4} - \mu\right)\times$$
$$\int_{\mathcal{L}'} \frac{\cosh kh}{k\sinh kh - K\cosh kh}\left(a_n\left[\frac{e^{ix\sqrt{k^2-k_0^2\sin^2\beta}}}{\sqrt{k^2-k_0^2\sin^2\beta}-\kappa_n} - \frac{e^{-ix\sqrt{k^2-k_0^2\sin^2\beta}}}{\sqrt{k^2-k_0^2\sin^2\beta}+\kappa_n}\right]\right.$$
$$\left. +b_n\left[\frac{e^{ix\sqrt{k^2-k_0^2\sin^2\beta}}}{\sqrt{k^2-k_0^2\sin^2\beta}+\kappa_n} - \frac{e^{-ix\sqrt{k^2-k_0^2\sin^2\beta}}}{\sqrt{k^2-k_0^2\sin^2\beta}-\kappa_n}\right]\right)\times$$
$$\frac{k\,dk}{\sqrt{k^2-k_0^2\sin^2\beta}} + Ae^{ik_0 x\cos\beta} \tag{23}$$

Here $\kappa^{(n)}$ is defined as

$$\kappa^{(n)} = \sqrt{\kappa_n^2 + k_0^2\sin^2\beta} \tag{24}$$

The coefficient b in expression (22) corresponds to $k_0\sin\beta$. The contribution of the integral along the branch cut must be zero. We assume that the poles at $\sqrt{k^2-k_0^2\sin^2\beta} = \kappa_n$ are in the upper half-plane and we apply the residue lemma at these points. Then dispersion relation for $\kappa^{(n)}$ for finite water depth follows

$$\left(\mathcal{D}\kappa^4 - \mu + 1\right)\kappa\tanh\kappa h = k_0 \tag{25}$$

which has such solutions in the complex plane: two at the real axis $\pm\kappa^{(1)}$, at the imaginary axis $\pm\kappa^{(n)}$, $n = 4, 5...$ and four in the complex plane $\pm\kappa^{(2)}$, $\pm\kappa^{(3)}$. For comparison of numerical results for different cases we are taking into account 3 roots (such as for next cases) which are physically realistic solutions for κ and are situated in the upper complex half-plane: real positive root $\kappa^{(1)}$ and two complex roots $\kappa^{(2)}$ and $\kappa^{(3)}$ with equal imaginary parts and equal but opposite-signed real parts.

We determine a value of critical angle of incidence. Angle becomes critical when κ_1 (corresponded value to real positive root $\kappa^{(1)}$ of dispersion relation) is equal 0, so

$$\sin\beta_{cr} = \kappa^{(1)}/k_0 \tag{26}$$

Now we consider the zeros of the dispersion relation for the water surface $k_0 \tanh k_0 h = K$. Our restriction to 3 roots implies that we take into account only one root of k_0 [1]. That leads us to the relations for the determination of the 'amplitudes' a_n and b_n. We insert (21) in (23) and obtain two linear relations:

$$\sum_{n=1}^{3} \left(\mathcal{D}\kappa^{(n)^4} - \mu\right) \frac{k_0 K}{\left(K(1-Kh) + k_0^2 h\right)} \left(\frac{a_n}{(\kappa_n - k_0 \cos\beta)\cos\beta} - \frac{b_n}{(\kappa_n + k_0 \cos\beta)\cos\beta}\right) + A = 0 \tag{27}$$

and

$$\sum_{n=1}^{3} \left(\mathcal{D}\kappa^{(n)^4} - \mu\right) \frac{k_0 K}{\left(K(1-Kh) + k_0^2 h\right)} \left(-\frac{a_n e^{i\kappa_n l}}{(\kappa_n + k_0 \cos\beta)\cos\beta} + \frac{b_n e^{-i\kappa_n l}}{(\kappa_n - k_0 \cos\beta)\cos\beta}\right) = 0 \tag{28}$$

Boundary conditions are the same for all values of depth. For the strip we get 4 equations from boundary conditions at both edges $x = 0$ and $x = l$. The zero moment condition leads to:

$$\sum_{n=1}^{3} \left(\kappa_n^2 + \nu k_0^2 \sin^2\beta\right)(a_n + b_n) = 0 \tag{29}$$

and

$$\sum_{n=1}^{3} \left(\kappa_n^2 + \nu k_0^2 \sin^2\beta\right)\left(a_n e^{i\kappa_n l} + b_n e^{-i\kappa_n l}\right) = 0 \tag{30}$$

The zero shear force condition leads to:

$$\sum_{n=1}^{3} \left(\kappa_n^3 + (2-\nu)\kappa_n k_0^2 \sin^2\beta\right)(a_n - b_n) = 0 \tag{31}$$

and

$$\sum_{n=1}^{3} \left(\kappa_n^3 + (2-\nu)\kappa_n k_0^2 \sin^2\beta\right)\left(a_n e^{i\kappa_n l} - b_n e^{-i\kappa_n l}\right) = 0 \tag{32}$$

Together with conditions (27) and (28) we have 6 equations. The solution of this system gives us values of the deflection components a_n and b_n. Reflection

[1] In principle more roots can be taken into account; in most cases the arrived accuracy is sufficient.

and transmission coefficients can be computed by adding up the two expressions in (14) and contribution of the pole $k = k_0$ in region $x < 0$ and $x > l$ respectively. We obtain for the reflection coefficient $|R|$ which is the amplitude of the reflected wave:

$$R = \frac{k_0 K}{\left(K - K^2 h + k_0^2 h\right)} \left(\sum_{n=1}^{3} \frac{\left(\mathcal{D}\kappa^{(n)^4} - \mu\right) a_n}{(\kappa_n + k_0 \cos\beta)\cos\beta} \left(e^{i(k_0+\kappa_n)l} - 1\right) + \sum_{n=1}^{3} \frac{\left(\mathcal{D}\kappa^{(n)^4} - \mu\right) b_n}{(\kappa_n - k_0 \cos\beta)\cos\beta} \left(e^{i(k_0-\kappa_n)l} - 1\right) \right) \tag{33}$$

and for the transmission coefficient T - the amplitude of the transmitted wave:

$$T = 1 + \frac{k_0 K}{\left(K - K^2 h + k_0^2 h\right)} \left(\sum_{n=1}^{3} \frac{\left(\mathcal{D}\kappa^{(n)^4} - \mu\right) a_n}{(\kappa_n - k_0 \cos\beta)\cos\beta} \left(e^{-i(k_0-\kappa_n)l} - 1\right) + \sum_{n=1}^{3} \frac{\left(\mathcal{D}\kappa^{(n)^4} - \mu\right) b_n}{(\kappa_n + k_0 \cos\beta)\cos\beta} \left(e^{-i(k_0+\kappa_n)l} - 1\right) \right) \tag{34}$$

Values of R and T will be determined for every case of depth and compared.

For the **semi-infinite** platform, the method described above is used whereas deflection is written in the form

$$w(x, y) = \sum_n a_n e^{i\kappa_n x + ik_0 y \sin\beta} \tag{35}$$

We are dealing with same Green's function, dispersion relation, κ_n, β_{cr}. Then we arrive to (27) without b_n-term and 2 boundary conditions at $x = 0$ which are

$$\sum_{n=1}^{3} \left(\kappa_n^2 + \nu k_0^2 \sin^2\beta\right) a_n = 0 \tag{36}$$

and

$$\sum_{n=1}^{3} \left(\kappa_n^3 + (2 - \nu)\kappa_n k_0^2 \sin^2\beta\right) a_n = 0 \tag{37}$$

After solving the system of 3 equations we obtain 3 components of the deflection. R_∞ - reflection coefficient for the semi-infinite platform can be computed by (33) without b_n-term.

When $\beta \geq \beta_{cr}$ our model remains valid. In this case we have three evanescent modes. $R = 1$ and $T = 0$ and the platform is deflected only near the edge $x = 0$.

Finite water depth case is general case. By taking the limit $h \to \infty$ we arrive to IWD case and the limit $h \to 0$ - to SWD case including transition from dispersion relation (25) for finite water to dispersion relations for infinite and shallow water respectively.

4 Infinite Water Depth

In this chapter the solution will be derived for infinite water. The way to obtain this solution will be based on the previous chapter but here we have some specialities. At first, wave number and potential of incoming waves for infinite water that is shown in (10) are different from FWD case.

By analogy way we split up fluid region, introduce the Green's function and obtain the expressions (16), (17) and (18) but with k_0 except the K. Finally we obtain an integro-differential equation for the deflection of the platform on infinite water:

$$\left(\mathcal{D}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2-\mu+1\right)w(x,y)=$$
$$\frac{k_0}{4\pi}\int_{\mathcal{P}}\left(\mathcal{D}\left(\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \eta^2}\right)^2-\mu\right)w(\xi,\eta)\mathcal{G}(x,y,\xi,\eta)\,d\xi d\eta+\phi_z^{inc} \tag{38}$$

The Green's function obeys the boundary condition at the free surface and radiation condition, see WEHAUSEN and LAITONE [9]

$$\mathcal{G}(x,y;\xi,\eta)=-2\int_{\mathcal{L}'}\frac{k}{k-k_0}J_0(kr)\,dk \tag{39}$$

at $z=0$, where $\mathcal{L}'$ and $J_0(kr)$ are described in previous chapter.

Case of the **strip**. The deflection of the platform is described by (21) with $n=3$ for same 'amplitudes' a_n, b_n, reduced wave numbers κ_n and β_{cr}.

If we insert the Green's function to (38) and integrate with respect to ξ and η then we obtain

$$\sum_n\left(\mathcal{D}\kappa^{(n)^4}-\mu+1)\right)\left(a_n e^{i\kappa_n x}+b_n e^{-i\kappa_n x}\right)=i\sum_n\frac{k_0}{2\pi}\left(\mathcal{D}\kappa^{(n)^4}-\mu\right)\times$$
$$\int_{\mathcal{L}'}\frac{k}{(k-k_0)\sqrt{k^2-k_0^2\sin^2\beta}}\left(a_n\left[\frac{e^{ix\sqrt{k^2-k_0^2\sin^2\beta}}}{\sqrt{k^2-k_0^2\sin^2\beta}-\kappa_n}-\frac{e^{-ix\sqrt{k^2-k_0^2\sin^2\beta}}}{\sqrt{k^2-k_0^2\sin^2\beta}+\kappa_n}\right]\right.$$
$$\left.+b_n\left[\frac{e^{ix\sqrt{k^2-k_0^2\sin^2\beta}}}{\sqrt{k^2-k_0^2\sin^2\beta}+\kappa_n}-\frac{e^{-ix\sqrt{k^2-k_0^2\sin^2\beta}}}{\sqrt{k^2-k_0^2\sin^2\beta}-\kappa_n}\right]\right)dk+Ae^{ik_0x\cos\beta} \tag{40}$$

Here $\kappa^{(n)}$ are roots of dispersion relation which is written for infinite water as

$$\Big(\mathcal{D}\kappa^4 - \mu + 1\Big)\kappa = \pm k_0 \tag{41}$$

We take into account 3 physically realistic roots for κ and they are same with real root $\kappa^{(1)}$ and two complex roots $\kappa^{(2)}$, $\kappa^{(3)}$ from finite water case. Here we neglect the contribution along the imaginary axis of the k-plane, after closing the contour of integration.

We consider the zeros of the dispersion relation for the water surface and insert (21) in (40) and obtain two linear relations:

$$\sum_{n=1}^{3}\Big(\mathcal{D}\kappa^{(n)^4} - \mu\Big)k_0\left(\frac{a_n}{(\kappa_n - k_0\cos\beta)\cos\beta} - \frac{b_n}{(\kappa_n + k_0\cos\beta)\cos\beta}\right) + A = 0 \tag{42}$$

and

$$\sum_{n=1}^{3}\Big(\mathcal{D}\kappa^{(n)^4} - \mu\Big)k_0\left(-\frac{a_n e^{i\kappa_n l}}{(\kappa_n + k_0\cos\beta)\cos\beta} + \frac{b_n e^{-i\kappa_n l}}{(\kappa_n - k_0\cos\beta)\cos\beta}\right) = 0 \tag{43}$$

Together with boundary conditions (29)-(32) we have 6 equations for finding the 'amplitudes' a_n and b_n. The solution of this system gives us values of the deflection. Reflection and transmission coefficients represented by such formulas:

$$\begin{aligned} R = &\sum_{n=1}^{3}\frac{\Big(\mathcal{D}\kappa^{(n)^4} - \mu\Big)k_0 a_n}{(\kappa_n + k_0\cos\beta)\cos\beta}\Big(e^{i(k_0+\kappa_n)l} - 1\Big) + \\ &\sum_{n=1}^{3}\frac{\Big(\mathcal{D}\kappa^{(n)^4} - \mu\Big)k_0 b_n}{(\kappa_n - k_0\cos\beta)\cos\beta}\Big(e^{i(k_0-\kappa_n)l} - 1\Big) \end{aligned} \tag{44}$$

and

$$\begin{aligned} T = 1 + &\sum_{n=1}^{3}\frac{\Big(\mathcal{D}\kappa^{(n)^4} - \mu\Big)k_0 a_n}{(\kappa_n - k_0\cos\beta)\cos\beta}\Big(e^{-i(k_0-\kappa_n)l} - 1\Big) + \\ &\sum_{n=1}^{3}\frac{\Big(\mathcal{D}\kappa^{(n)^4} - \mu\Big)k_0 b_n}{(\kappa_n + k_0\cos\beta)\cos\beta}\Big(e^{-i(k_0+\kappa_n)l} - 1\Big) \end{aligned} \tag{45}$$

For the **semi-infinite** platform deflection is written in form (35). We arrive by same method to (42) without b_n-term and 2 boundary conditions at $x = 0$ (36) and (37). Reflection coefficient can be computed by (44) without b_n-term.

5 Shallow Water Depth

Here the solution will be derived for shallow water. It will be based on transition conditions and that is difference from FWD and IWD cases.

For the platform which floats on fluid of shallow depth in accordance with the derivation of the shallow water theory by STOKER [10] we have the approximation condition

$$\Phi_z = -h\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi \tag{46}$$

The harmonic wave is written as (9) and then from (3) by using of (46) and (9) can be obtained

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi - k_0^2\phi = 0,\ x, y \in \mathcal{F} \tag{47}$$

and

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi - \frac{i\omega}{h}w = 0,\ x, y \in \mathcal{P} \tag{48}$$

The general solution of (47) and the conditions at ∞ leads to:

$$\phi(x, y) = Be^{ik_0(x\cos\beta + y\sin\beta)} + Re^{-ik_0 x\cos\beta + ik_0 y\sin\beta},\ x < 0 \tag{49}$$

where the first term represent a progressed wave moving to the right and second - reflected wave moving to the left.

$$\phi(x, y) = Te^{ik_0(x\cos\beta + y\sin\beta)},\ x > l \tag{50}$$

where right part represent the transmitted wave.

Insertion of condition (5) to equation (4) and using of (48) leads us to the differential equation for $\phi^{\mathcal{P}}$ - total potential under the platform:

$$\mathcal{D}\Delta^3\phi^{\mathcal{P}} + (1-\mu)\Delta\phi^{\mathcal{P}} - k_0^2\phi^{\mathcal{P}} = 0,\ x, y \in \mathcal{P} \tag{51}$$

For the open fluid surface we have (47).

Transition conditions written in the following form

$$\phi^{\mathcal{P}} = \phi^{\mathcal{F}},\ \frac{\partial\phi^{\mathcal{P}}}{\partial n} = \frac{\partial\phi^{\mathcal{F}}}{\partial n},\ x, y \in \mathcal{S} \tag{52}$$

Deflection is written in form (21) for same with previous chapters a_n, b_n, κ_n and β_{cr}. For shallow water case the connection between platform deflection and potential described by (48). Dispersion relation is

$$\left(\mathcal{D}\kappa^4 - \mu + 1\right)\kappa^2 = k_0^2 \tag{53}$$

We use three roots which lies in the upper complex half-plane.

Reflection R and transmission T coefficients will be find at once with the deflection components. Such way for the definition of 6 components of the deflection and coefficients of reflection and transmission we need system of 8 equations. We have already 4 from boundary conditions (29)-(32). Rest 4 equations can be obtained from transition conditions (52) at both edges of the platform. Due to (49) and (50) we have at $x = 0$:

$$1 + R = -\frac{\omega}{ih}\sum_{n=1}^{3}(a_n + b_n) \tag{54}$$

and

$$ik_0 \cos\beta(1 - R) = -\frac{\omega}{ih}\sum_{n=1}^{3} i\kappa_n(a_n - b_n) \tag{55}$$

and at $x = l$:

$$Te^{ik_0 l\cos\beta} = -\frac{\omega}{ih}\sum_{n=1}^{3}\left(a_n e^{i\kappa_n l} + b_n e^{-i\kappa_n l}\right) \tag{56}$$

and

$$ik_0 \cos\beta Te^{ik_0 l\cos\beta} = -\frac{\omega}{ih}\sum_{n=1}^{3} i\kappa_n\left(a_n e^{i\kappa_n l} - b_n e^{-i\kappa_n l}\right) \tag{57}$$

We obtain values of w, R and T after solving of the system (29)-(32), (54)-(57).

Deflection and reflection coefficient for the SIP can be obtained by same way also. Then deflection presented as (35). From transition condition (52) and (49) we obtain conditions (54)-(55) at the edge $x = 0$ without b_n-terms. Rest 2 equations are boundary conditions (36)-(37). After solving of the system we obtain 3 components of the deflection and reflection coefficient R_∞ for the SIP.

6 Results. Their Comparison and Discussion

We give results for the deflection of the strip and of the semi-infinite platform. In figures 3-5 we show the results for both forms of the platform for different values of angle for IWD, for FWD and for SWD respectively.

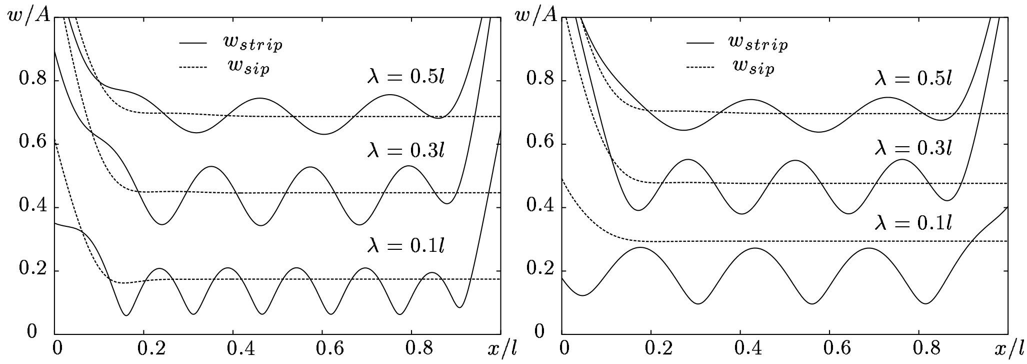

Fig.3 Comparison of the results for the deflection w for the strip and SIP for $D/\rho g = 10^5\mathrm{m}^4$, for $\beta = 0°$ (a) and for $\beta = 15°$ (b) for IWD

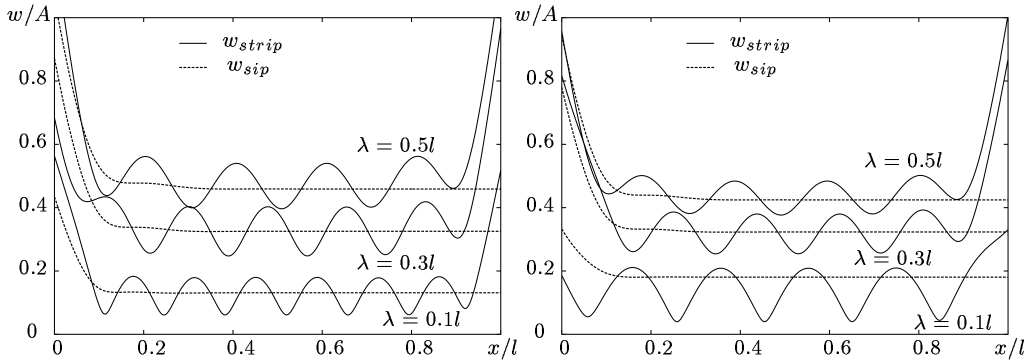

Fig.4 Comparison of the results for the deflection w for the strip and SIP for $D/\rho g = 10^5\mathrm{m}^4$, for $\beta = 0°$ (a) and for $\beta = 15°$ (b) for FWD ($h = 10\mathrm{m}$)

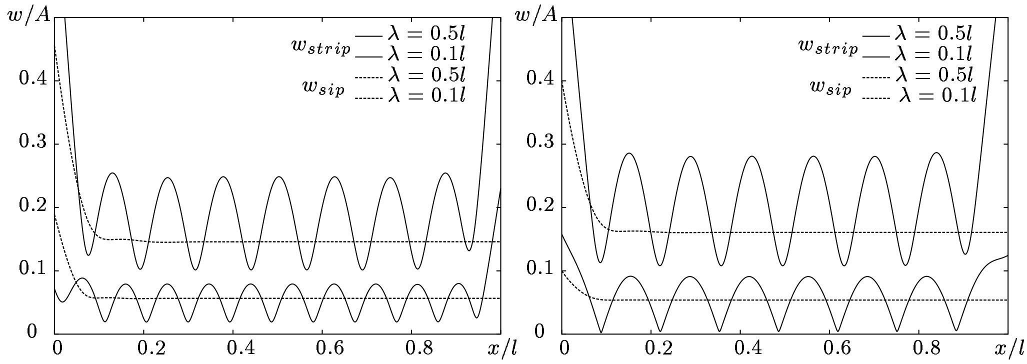

Fig.5 Comparison of the results for the deflection w for the strip and SIP for $D/\rho g = 10^5\mathrm{m}^4$; $\beta = 0°$ in (a) and $\beta = 10°$ in (b) for SWD ($h = 1\mathrm{m}$)

In figure 6 we show comparison of the results for the strip deflection for different wavelengths. Results are shown for three values of depth for zero angle of incidence.

In figures 7-8 we compare the results for the deflection of the strip and of the semi-infinite platform for same wavelength and depth for FWD and for IWD cases. For each form of the platform results are presented for 4 different values of incident angle.

In figure 9 we compare results for flexural rigidity $D/\rho g = 10^7\mathrm{m}^4$ for IWD and

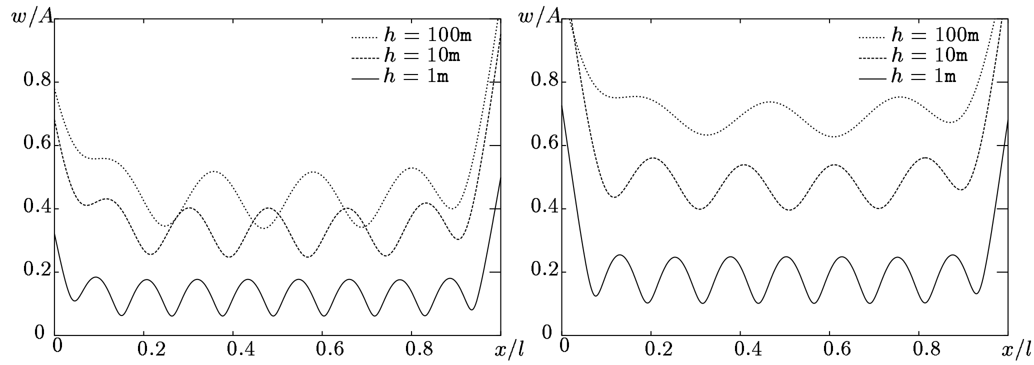

Fig.6 Results for the strip deflection for $D/\rho g = 10^5\mathrm{m}^4$, $h = 1, 10, 100\mathrm{m}$ for $\lambda = 0.3l$ (a) and $\lambda = 0.5l$ (b)

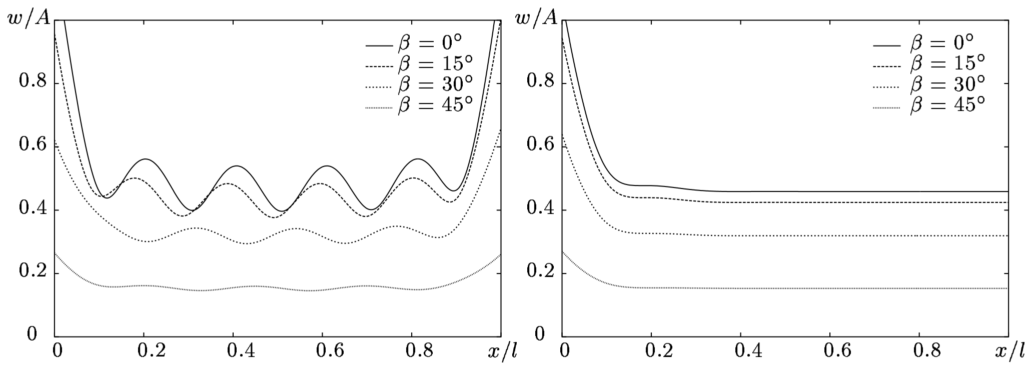

Fig.7 Results for the deflection for $D/\rho g = 10^5\mathrm{m}^4$, $h = 10\mathrm{m}$, $\lambda = 0.5l$ for the strip (a) and for the SIP (b) (finite water depth)

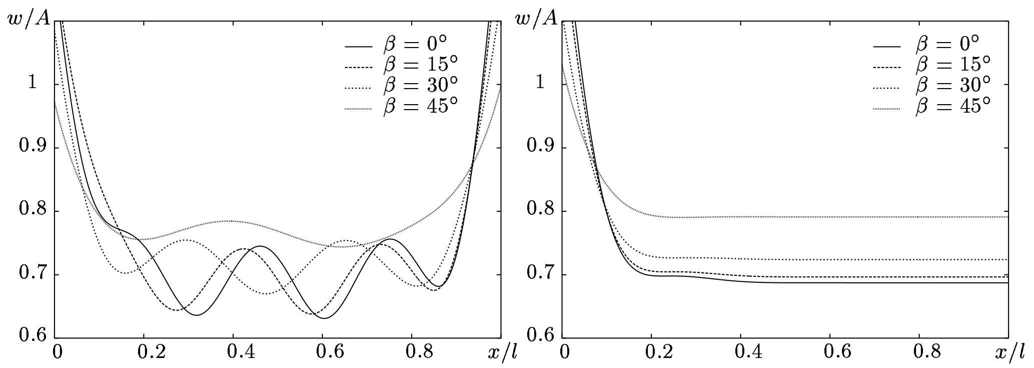

Fig.8 Results for the deflection for $D/\rho g = 10^5\mathrm{m}^4$, $\lambda = 0.5l$ for the strip (a) and for the SIP (b) (infinite water depth)

FWD cases.

In figure 10 we show results for finite water for different values of β including $\beta \geq \beta_{cr}$. As we note before in such cases our model remains valid.

In figure 11 we show the results for reflection and transmission coefficients for FWD and for SWD cases. As we can see, the peaks of the reflection coefficient for SWD much higher then for FWD which are quite close to graphic for IWD case. Wave energy is conserved, up to a high degree of accuracy $|R|^2+|T|^2 = 1$.

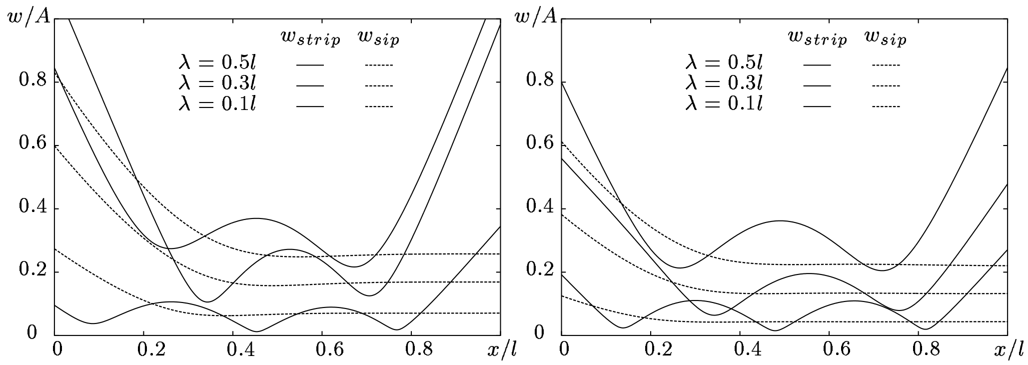

Fig.9 Comparison of the results for the deflection for the strip and SIP for $D/\rho g = 10^7\mathtt{m}^4$, $\beta = 0°$ on infinite (a) and finite ($h = 100\mathtt{m}$) (b) water

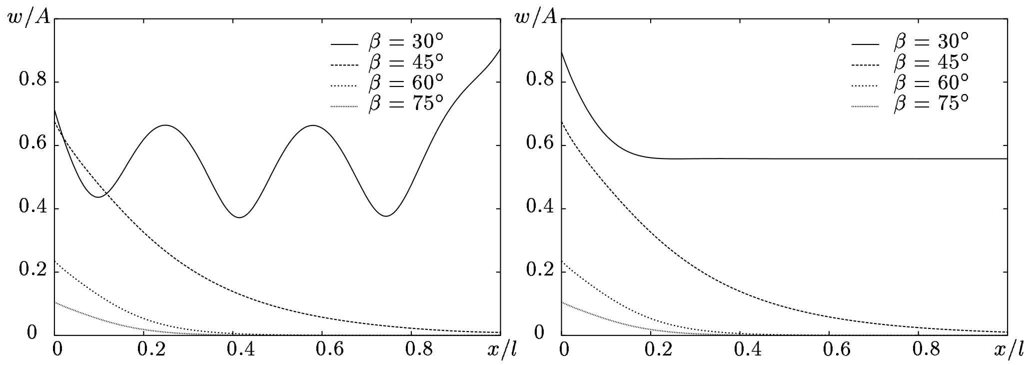

Fig.10 Results for the deflection for $D/\rho g = 10^5\mathtt{m}^4$, $h = 100\mathtt{m}$, $\lambda = 0.3l$ for the strip (a) and for the SIP (b) (finite water depth)

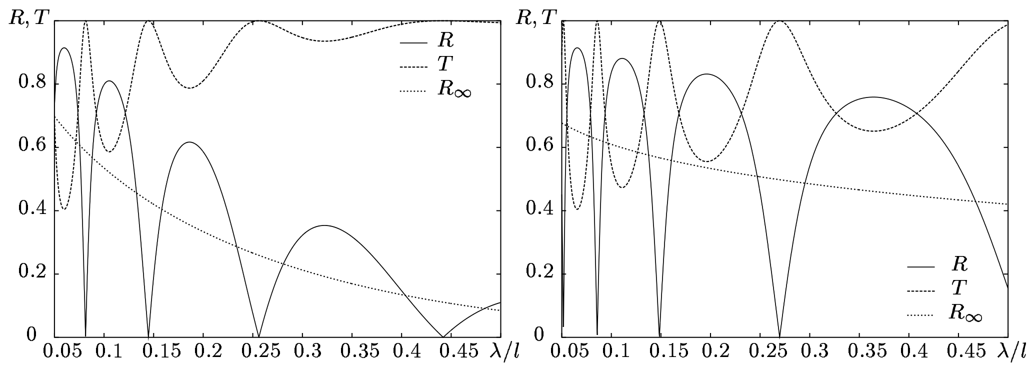

Fig.11 Reflection and transmission coefficients for $D/\rho g = 10^5\mathtt{m}^4$, $\beta = 0°$ for $h = 100\mathtt{m}$ (a) and $h = 1\mathtt{m}$ (b)

In figure 12 we demonstrate the influence of the number of roots of dispersion relation. Results are compared for 3 values of wavelength. In the lower graph in 12a the wave depth can be considered to be infinite (wavelength is 30m while depth is 200m). The difference between results is large near the edges of the strip. All other graphs correspond to the finite depth case. Here the differences of the results are small, especially if we consider the case of small depth.

In [3] Hermans compared for the finite depth case the results obtained by presented approach with the results of Takagi et al. [4] who used an eigen-

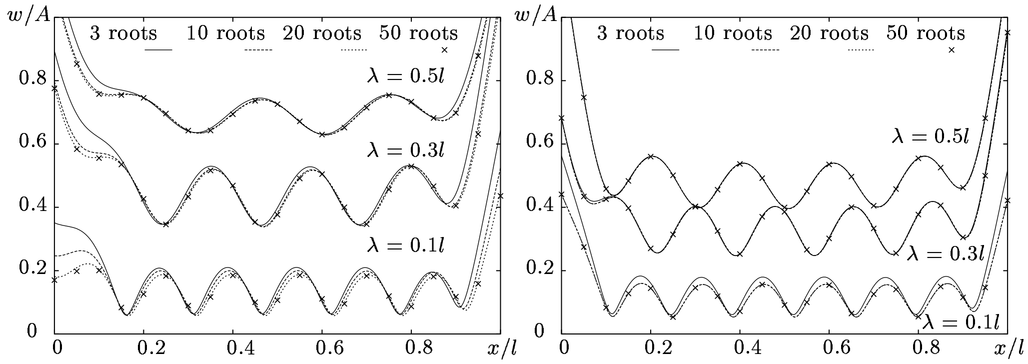

Fig.12 Comparison of the results for $D/\rho g = 10^5\mathtt{m}^4$, $\beta = 0°$ for 3, 10, 20 and 50 wave modes, $h = 200\mathtt{m}$ (a) and $h = 10\mathtt{m}$ (b)

function expansion method. For case of the infinite depth, in [2], numerical results were compared with the results obtained by using the Wiener-Hopf technique in TKACHEVA [6]. In the same paper a comparison is presented with the results obtained by boundary element computations in HERMANS [1]. Comparison proves accuracy of present method.

7 Conclusions

We have shown results for three different situations. In the IWD case and the FWD case the results are obtained by means of an analysis of the integro-differential equation. For comparison in each method three modes of the deflection are considered, one traveling and two evanescent modes. In the SWD case major simplification is the shallow water approximation. From our numerical results it is shown that all these approximations are consistent with each other. In the transition from one case to the other the numerical values show a continuous behavior.

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