Hydroelastic Behaviour of a Ring-Shaped Plate

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The problem of diffraction of water waves on a very large floating platform is considered. We study the hydroelastic behaviour of a ring-shaped floating elastic plate and its response to incident surface waves. The thin plate with elastic properties is a model of the floating platform due to its small thickness and shallow draft.

The ring floats at the surface of ideal, incompressible fluid. The wavelength is less than the diameter of the gap in the ring. We consider two cases: the first one is general with finite water depth, and the second is a deep water case where the depth is assumed to be infinite. We use an integro-differential formulation [1,2] which allows us to solve this problem for any shape of the floating plate. Also the thin plate theory, standard Laplace equation in the fluid, supplemented with the surface and boundary conditions, are used.

The plate deflection is represented as a series of Hankel functions with corresponding coefficients in the following form:

$$w(\rho,\varphi) = \sum_{m=1}^{M}\sum_{n=0}^{N}\left[a_{mn}H_n^{(1)}\left(\kappa_m\rho\right) + b_{mn}H_n^{(2)}\left(\kappa_m\rho\right)\right]\cos n\varphi, \qquad (1)$$

where a_{mn} and b_{mn} are the unknown amplitude functions, and M is the number of κ_m - roots of the dispersion relation in the plate region taken into account. In a similar way, we represent the Green function, obeying the boundary conditions at the free surface, for both depth cases as a series of Bessel functions. Later, Graf's addition theorem is applied to the Green function.

Then, we derive the governing integro-differential equation for the problem. An analysis of integrals in the complex plane and use of Cauchy's theorem and the Wronskian leads us to the dispersion relation. Finally, we obtain a set of integro-differential equations, which, together with free edge conditions, allows us to find the unknown amplitudes of the plate deflection. Also we study the free surface elevation in the ring gap.

Thus, for the deflection of a thin elastic ring floating at the free surface of the water with finite depth, an exact analytical solution has been obtained. Numerical results are obtained for different values of the physical parameters. The approach, described above, can be applied to the plates of other rotational symmetric configurations.

References

[1] Andrianov AI, Hermans AJ. The Influence of Water Depth on the Hydroelastic Response of a Very Large Floating Platform. *Marine Structures* 2003; 16 (5): p.355-71.

[2] Andrianov AI, Hermans AJ. Hydroelasticity of a Circular Plate on Water of Finite or Infinite Depth. *Journal of Fluids and Structures, submitted.*