Initiated Wave Field

Initiated Wave Field (Pattern).

Free surface displacement generated by plate motion.

General description.

Free surface elevation in the open fluid $\mathcal{F}$ equals to a sum of the incident wave elevation and the additional wave elevation, generated by the plate motion

$$\zeta = \zeta^{inc} + \zeta^{pm}. \tag{1}$$

The total potential in $\mathcal{F}$ is also represented as a sum of the incident wave potential and the potential of waves, arising from the plate presence

$$\phi^{\mathcal{F}} = \phi^{inc} + \phi^{pm}, \tag{2}$$

here ϕ^{pf} is the classical diffraction potential plus radiation potential.

In polar coordinates

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} + \frac{K}{4\pi}\int\limits_{\mathcal{P}} \left\{\mathcal{D}\Delta^2 - \mu\right\} w(r,\theta)\,\mathcal{G}(\rho,\varphi,0;r,\theta,0)\,dS. \tag{3}$$

$dS = r\,dr\,d\theta$

Circle

$\mathcal{F}$: $r > r_0$.

IWD

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - 2\pi i r_0 \sum_{m=1}^{M} \frac{k_0^2}{(k_0^2 - \kappa_m^2)} \left(\mathcal{D}\kappa_m^4 - \mu\right)$$
$$\times \sum_{n=0}^{N} a_{mn}\left[k_0 J_{n+1}(k_0 r_0) J_n(\kappa_m r_0) - \kappa_m J_n(k_0 r_0) J_{n+1}(\kappa_m r_0)\right] J_n(k_0\rho)dk. \tag{4}$$

FWD

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - 2\pi i K r_0 \sum_{m=1}^{M}\sum_{i=0}^{M-3} \frac{k_i^2}{(k_i^2 - \kappa_m^2)(k_i^2 h - K^2 h + K)} \left(\mathcal{D}\kappa_m^4 - \mu\right)$$
$$\times \sum_{n=0}^{N} a_{mn}\left[k_i J_{n+1}(k_i r_0) J_n(\kappa_m r_0) - \kappa_m J_n(k_i r_0) J_{n+1}(\kappa_m r_0)\right] J_n(k_i\rho)dk. \tag{5}$$

Ring

Free surface elevation in the open fluid $\mathcal{F}_0$ ($r > r_0$) and gap $\mathcal{F}_1$ ($r < r_1$) regions. We continue the analysis of plate-water interaction and consider open fluid inside of the ring $\mathcal{F}_1$. The elevation $\zeta(\rho,\varphi)$ of the free surface in the gap can be computed by (1), where the value of w^{inc} may be obtained from incident wave potential expression with use of kinematic condition and the value of w^{pf} - from analysis of IDE in the gap area.

If we are dealing with FWD case, the free surface elevation in the gap, after use of residue lemma at th poles $k = k_i$, takes the form

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - K\int_{-\infty}^{\infty}\sum_{m=1}^{M}\left(\mathcal{D}\kappa_m^4 - \mu\right)\frac{J_n(k\rho)}{(k^2-\kappa_m^2)}$$
$$\times\sum_{i=0}^{M-3}\frac{k_i^2}{(k_i^2h - K^2h + K)(k-k_i)}\left[a_{mn}c_{mn}^{(1)} + b_{mn}c_{mn}^{(2)}\right]dk. \tag{6}$$

As we are at the gap area $\mathcal{F}_1$ $\rho < r_1 < r_0$. Therefore, the contour of the integration can be closed by splitting up of Bessel functions $J_t(kr_i)$, $t = n, n+1$, $i = 0, 1$, into half-sums of corresponding Hankel functions. Finally, for the elevation in the gap we obtained the following expression

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - 2\pi i\sum_{m=1}^{M}\left(\mathcal{D}\kappa_m^4 - \mu\right)$$
$$\times\sum_{i=0}^{M-3}\frac{k_i^2K}{(k_i^2h - K^2h + K)}\frac{J_n(k_i\rho)}{(k_i^2-\kappa_m^2)}\left[a_{mn}f_{mni}^{(1)} + b_{mn}f_{mni}^{(2)}\right]dk, \tag{7}$$

where introduced functions $f_{mni}^{(q)}$ are

$$f_{mni}^{(q)} = r_0\left[kH_{n+1}^{(1)}(k_ir_0)H_n^{(q)}(\kappa_mr_0) - \kappa_mH_n^{(1)}(k_ir_0)H_{n+1}^{(q)}(\kappa_mr_0)\right] -$$
$$r_1\left[kH_{n+1}^{(1)}(k_ir_1)H_n^{(q)}(\kappa_mr_1) - \kappa_mH_n^{(1)}(k_ir_1)H_{n+1}^{(q)}(\kappa_mr_1)\right], \quad q = 1,2. \tag{8}$$

In general, for infinitely deep water the procedure is the same, but with the pole $k = k_0$ only. We derive the following equation for the elevation

$$\zeta(\rho,\varphi) = Ae^{ik_0\rho\cos\varphi} - 2\pi i\sum_{m=1}^{M}\left(\mathcal{D}\kappa_m^4 - \mu\right)\frac{k_0^2J_n(k_0\rho)}{(k_0^2-\kappa_m^2)}\left[a_{mn}f_{mn0}^{(1)} + b_{mn}f_{mn0}^{(2)}\right]dk. \tag{9}$$